MOTION OF CHARGED PARTICLES IN ELECTRIC & MAGNETIC FIELDS

BSc – I (UNIT III)
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**Basic Definitions**

**Electric Field:** An electric field is said to exist in a region of space if an electric charge, at rest, experiences a force of electric origin. The electric field strength or intensity \( E \) at a point in space is defined as the amount of force \( F \) experienced by a unit positive charge \( q \) placed at that point.

\[
E = \frac{F}{q}
\]

Electric field between two parallel plates having a potential difference of \( V \) volts and having separation \( d \) metres between them is given as

\[
E = \frac{V}{d}
\]

The SI unit for Electric field is Newtons/Coulomb (N/C).

**Electric Potential:** Electric potential \( V \) at a point is defined as the work done on a positive charge to bring it from infinity to that point.

\[
V = \frac{W}{q}
\]

The SI unit for Electric potential is Volts (V) or Joules/Coulomb (J/C).

**Magnetic Field:** Moving charges experience an additional interaction in addition to the Electrostatic interaction. This interaction is known as magnetic interaction. The field by which the magnetic interaction occurs is called the magnetic field and is characterized by the magnetic induction vector \( B \). The SI unit for magnetic induction vector is Tesla (T) or Webers/metre\(^2\) (Wb/m\(^2\)). In CGS, the unit is Gauss (G). \( 1 \text{T} = 10^4 \text{G} \).

Magnetic field is characterized by another vector \( H \) and is called as the magnetic field strength. It is defined as the ratio of magnetic induction in vacuum to the permeability \( \mu_0 \).

\[
H = \frac{B}{\mu_0}
\]

SI unit for magnetic field strength is Ampere-turn/metre (A/m). CGS unit is oersted (Oe). \( 1 \text{Oe} = 80 \text{A/m} \).

**Force on a current carrying conductor in a magnetic field:**

Consider a conductor of length \( l \), carrying current \( I \) and is placed in a magnetic field of flux density \( B \) then the magnitude of the force is given as,

\[
F = BIl\sin\theta
\]

Where, \( \theta = \) Angle between the direction of current and the direction of the magnetic field.

The direction of the force is given by Fleming’s Left Hand Rule.

As per this rule, stretch three fingers of left hand mutually perpendicular to each other. The index finger points in the direction of magnetic field, the middle finger points towards the direction of current and the thumb gives the direction of the force.
**Force on a moving charge in a magnetic field:**

Consider a charge $q$ moving with velocity $\mathbf{v}$ in a magnetic field $\mathbf{B}$, the force acting on the charge is given by

$$F = Bq \sin \theta$$

Where, $\theta$ = Angle between the direction of motion of charge and the direction of the magnetic field.

The direction of force is given by the Fleming’s left hand rule.

![Diagram of force on a moving charge in a magnetic field]

The trajectories for positively and negatively charged particles in a magnetic field can be shown as

**Lorentz Force:** The force on a point charge due to electromagnetic fields. It is given by the following equation

$$F = q (\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

**Motion of Charged Particle in a Uniform Electric Field:**

1) **Motion Parallel to Electric Field:**

Assume two parallel plates in vacuum separated by a distance $d$ metres as shown below:

![Diagram of motion parallel to electric field]

A potential difference of $V$ Volts is applied between them. A uniform electric field $E$ is developed in the region between the plates given by $E = V/d$.

A free electron of mass $m$ and charge $q$ is placed at rest in the uniform electric field and released. The electron experiences a force due to the electric field given by

$$F = -qE$$

Also, according to Newton’s second law of motion,

$$F = ma$$
Where, \( m \) is the mass of the particle and \( a \) is the acceleration.

Hence, in our case acceleration of the electron in the electric field is given by,

\[
a = \frac{F}{m} = \frac{-qE}{m} = \frac{-qV}{dm}
\]

The negative sign signifies that the force \( F \) accelerates the electron in a direction opposite to electric field \( E \). (We will drop the negative sign for calculation purposes and take absolute values.)

We can apply laws of kinematics and find the other quantities.

Taking the initial conditions,

Initial velocity, \( u = 0 \)

Initial displacement, \( S_0 = 0 \).

Therefore, velocity of the electron at any given time \( t \) is given as:

\[
v = u + at = \frac{qEt}{m} = \frac{qVt}{md}
\]

And the distance travelled by the electron during time \( t \) is given as:

\[
S = ut + \frac{1}{2}at^2 = \frac{qEt^2}{2m} = \frac{qVt^2}{2md}
\]

Also,

\[
v^2 = u^2 + 2aS = \frac{2qES}{m} = \frac{2qVS}{md}
\]

Time required by electron to travel distance \( S \) is given by:

\[
t = \sqrt{\frac{2mS}{qE}} = \sqrt{\frac{2mdS}{qV}}
\]

Kinetic energy attained by the electron after moving a distance \( S \) is given by:

\[
K = \frac{1}{2} mv^2 = \left( \frac{1}{2} m \right) \left( \frac{2qES}{m} \right) = qES = \frac{qVS}{d}
\]

The electron gains energy from the electric field which it converts to kinetic energy and hence

\[
\frac{1}{2} mv^2 = qV
\]

Therefore,

\[
v = \sqrt{\frac{2qV}{m}}
\]

This velocity is called as impact velocity or terminal velocity.

Putting values of \( m \) and \( q \) for an electron, we find

\[
v = 5.93 \sqrt{V} \times 10^5 \text{ m/s}
\]

An electron volt (eV) is the energy acquired by an electron when it is accelerated through a potential of one volt.

\[
1 \text{ eV} = 1.6 \times 10^{-19} \text{J}
\]
2) **Motion Perpendicular to Electric Field (Electric Field As An Accelerating Field):**

Consider two parallel plates of length $l$ having a potential difference $V$ in between them. The plates are at a distance $d$ from each other. The electric field ($E$) is directed from the positive to the negative plate. An electron having charge $q$, mass $m$ and initial velocity in the x-direction ($v_x$) enters the space between the plates having direction perpendicular to that of the electric field. This is as shown below.

![Diagram of electron motion in electric field](image)

The electron is only attracted towards the positive plate and thus its acceleration has only y-component. After emerging from the field the electron will travel along a straight line and strike the screen.

The vertical acceleration $a_y$ is given by:

$$a_y = \frac{F}{m} = \frac{qE}{m} = \frac{qV}{md}$$

The displacement of electron in the x-direction is given by:

$$x = v_x t$$

Thus, the time $t$ for which the electron remains in the electric field between the plates is given by:

$$t = \frac{x}{v_x}$$

The constant vertical velocity of electron while leaving the field is given by:

$$v_y = u_y + a_y t$$

$$v_y = \frac{qEt}{m} = \frac{qVt}{md}$$

The displacement in the y-direction inside the electric field is given by:

$$y_p = \frac{1}{2} a_y t^2 = \frac{qE}{2m} t^2 = \frac{qV}{2md} t^2$$

Eliminating $t$ from the above two equations we get

$$y_p = \frac{qE}{2m v_x^2} x^2 = \frac{qV}{2md v_x^2} x^2$$

Thus,

$$y_p = k x^2$$

Where,

$$k = \frac{qE}{2m v_x^2} = \frac{qV}{2md v_x^2}$$

The relationship $y_p = k x^2$ represents a parabola. This implies that an electron travels along a parabolic path in a transverse electric field.
**Deflection Of The Charged Particle By Transverse Electric Field (Electric Field As Deflecting Field):**

In the region outside the electric field, the deflecting force no longer acts on the electron and it travels in a straight line. The velocity $v$ is the resultant of $v_x$ and $v_y$.

$$v = \sqrt{v_x^2 + v_y^2}$$

The straight line is extended backwards to meet the x-axis. This line makes an angle $\theta$ with the x-axis given by

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right)$$

The vertical deflection is given by

$$y_E = D \tan \theta = D \frac{v_y}{v_x}$$

Using $x = l$, we get

$$v_y = \frac{qE}{mv_x} = \frac{qvl}{mdv_x}$$

$$y_E = \frac{DqE}{mv_x^2} = \frac{Dqvl}{mdv_x^2}$$

Before entering the plates, if the electron has been accelerated by a potential difference $V_A$ then

$$\frac{1}{2}mv_x^2 = qV_A$$

$$v_x^2 = \frac{2qV_A}{m}$$

Therefore, the vertical displacement $y$ is given by

$$y_E = \frac{1}{2} \frac{Dlv}{2dV_A}$$

**Transit time:** The time $t$ spent by the electron in the electric field is called the transit time of the electron. It is given as

$$t = \frac{l}{v_x}$$

**Deflection Sensitivity:** It is defined as the deflection caused by one volt of potential difference applied to the deflecting plates.

It is thus,

$$S = \frac{y_E}{V} = \frac{1D}{2dV_A}$$

The reciprocal of deflection sensitivity is called the deflection factor.

Thus,

$$G = \frac{2dV_A}{1D}$$

The deflection factor is expressed in Volts/cm.
Motion of Charged Particle in a Uniform Magnetic Field:

A static magnetic field does not exert any force on a charged particle at rest. It can experience a magnetic force only when it enters the magnetic field with some velocity $v$. The force is given as

$$F_L = q \left( v \times B \right) = qvB \sin \theta$$

The direction of the force is along the normal to the plane containing the vectors $v$ and $B$. This means that the Lorentz force is always perpendicular to the displacement of the electron.

If $dx$ is the infinitesimal displacement of electron during the time interval $dt$, the work done $dW$ on electron by the magnetic field is given by

$$dW = F_L \cdot dx = F_L \cdot vdt = q(v \times B) \cdot vdt = 0$$

It implies that a steady magnetic field does no work when the charged particle is displaced. This means that the kinetic energy of the charged particle does not change due to the action of the magnetic field.

As the force $F_L$ and velocity $v$ are perpendicular to each other, we get

$$F_L \cdot v = mav \cos 90^\circ = 0$$

$$= m \frac{dv}{dt} v = d\left( \frac{v^2}{2} \right) = 0$$

$$\frac{1}{2}mv^2 = \text{constant} \quad \text{or} \quad v = \text{constant}$$

The applied magnetic field can only change the direction of the velocity vector but can’t alter the speed of the moving charged particle.

1) **Motion parallel to the Magnetic Field (Longitudinal Field):**

If a charged particle moves along the magnetic lines of induction, the Lorentz force is given by

$$F_L = qvB \sin \theta = qvB \sin 0 = 0$$

Similarly, if the charged particle moves opposite to the field lines then the Lorentz force is given by

$$F_L = qvB \sin \theta = qvB \sin 180 = 0$$

In both the cases the charged particle continues to move along the initial direction of motion without a change in speed or direction.
2) Motion perpendicular to the Magnetic Field (Transverse Field):

Let us now consider the case of an electron of mass \( m \) and charge \( q \) entering a uniform magnetic field \( B \) with its initial velocity vector \( v_x \) perpendicular to the field. The magnetic field is considered to be having a direction into the plane of the paper, indicated by the crosses. The magnetic force is given by

\[
F_L = qv_x B
\]

which has a constant magnitude. This force can’t change the magnitude of electron velocity but can deflect the electron continuously along a curvilinear path. The tangential component of \( F_L \) will be zero as \( F_L \) and velocity \( v_x \) are mutually perpendicular. Therefore, the normal component will be equal to \( F_L \) itself. This will always act perpendicular to \( v_x \) at each point. Therefore, \( F_L \) is a centripetal force. According to Newton’s laws of motion, centripetal force is given by

\[
F_c = \frac{mv_x^2}{r}
\]

Therefore,

\[
qv_x B = \frac{mv_x^2}{r}
\]

or,

\[
r = \frac{mv_x}{qB}
\]

Since all parameters are constant, \( r = \text{constant} \). Therefore, the path followed by the electron would be a circle. Thus, the electron will describe a circular path in a plane perpendicular to the magnetic induction lines. The sense of rotation will be clockwise if the magnetic field is directed into the plane of paper. For a positive charge the sense of rotation will be anti-clockwise.

The time period of one revolution is given by

\[
T = \frac{(Distance\ covered\ in\ one\ revolution)}{(Speed\ of\ the\ particle)}
\]

\[
T = \frac{2\pi r}{v_x} = \left(\frac{2\pi}{v_x}\right)\left(\frac{mv_x}{qB}\right) = \frac{2\pi m}{qB}
\]

The frequency of revolution of the object is given by

\[
f = \frac{1}{T} = \frac{qB}{2\pi m}
\]

This indicates that the time period and frequency are independent of the velocity of the electron and radius of the circular path. \( v_x \) and \( r \) adjust such that \( T \) and \( f \) stay constant.

Thus, slower particles move in smaller circles while faster particles move in larger circles so as to keep the time period constant.
3) **Motion of Charged Particle projected at an Angle into a Magnetic field:**

Let an electron of mass \( m \) and charge \( q \) with a uniform velocity \( v \) enter a uniform magnetic field \( B \) at an angle \( \theta \). Let us have the magnetic field in z-direction. Thus, the electron velocity can be resolved into rectangular components \( v_x \) and \( v_z \). The component of velocity parallel to the magnetic induction, \( v = v \cos \theta \), is not influenced by the field as

\[
F_{Lz} = q(v_z \times B) = 0
\]

Hence, the electron continues to travel along the field lines with a velocity \( v_z = v \cos \theta \).

The velocity component \( v_x = v \sin \theta \) gives rise to a force on the electron

\[
F_{Lx} = qv_x B = qvB\sin \theta
\]

Under the action of this force, the electron tends to describe a circular path in a plane perpendicular to the magnetic field. The radius of this path is given by:

\[
R = \frac{mv_x}{qB} = \frac{mv \sin \theta}{qB}
\]

Time period of one revolution is given by

\[
T = \frac{2\pi R}{v_x} = \left( \frac{2\pi}{v \sin \theta} \right) \left( \frac{mv \sin \theta}{qB} \right) = \frac{2\pi m}{qB}
\]

The resultant motion of the electron is obtained by superposition of the uniform translational motion parallel to \( B \) and the uniform circular motion in a plane normal to \( B \). The resultant motion is along a helical path with axis of the helix being the field direction.

The pitch of the helix is the distance covered by the electron along the field direction in one revolution. Thus, the pitch \( p \) is given by

\[
p = v_z T = (v \cos \theta)T
\]

\[
p = \frac{2\pi mv \cos \theta}{qB}
\]

**Magnetostatic Deflection**

Let us assume that a uniform magnetic field is formed over a length \( l \). It is assumed that the magnetic lines of force are directed into the plane of the paper. Let an electron beam having velocity \( v_x \) enter the magnetic field in a direction perpendicular to the field direction. The electron beam bends through an arc having radius \( r \). After emerging from the field, the beam continues to travel in a straight line and strikes the fluorescent screen at \( Q \). Let the deflection produced be \( y_B \) as shown in figure.
The circular arc subtends an angle $\theta$ at B. AB and BC are the radii of the circle having its centre at B. AO and OQ are the tangents to the arc AC at points A and C respectively. Therefore $\angle POQ = \theta$. The vertical deflection $PQ = y$ is given by

$$PQ = y = D \tan \theta$$

where, $D$ is the distance of the screen from the centre of the magnetic field. When $\theta$ is small, we can write,

$$y = D \left( \frac{AC}{r} \right) \approx \frac{Dl}{r}$$

We have,

$$r = \frac{mv_x}{qB}$$

Thus,

$$y = \frac{DlqB}{mv_x}$$

Also, $v_x = \sqrt{\frac{2qV_A}{m}}$

Hence,

$$y = DlB \sqrt{\frac{q}{2mV_A}}$$

The magnetic deflection sensitivity is given by:

$$S_M = \frac{y}{B} = Dl \sqrt{\frac{q}{2mV_A}}$$

Therefore, the deflection sensitivity is inversely proportional to the square root of the accelerating voltage, $V_A$.

**Velocity Selector:**

Velocity filter is an electro-optic device which uses uniform electric and magnetic field in crossed field configuration (perpendicular to each other) for setting a stream of charged particles of single velocity from a beam of particles having a wide range of velocities.

The electric field deflects the electrons upwards while the magnetic field deflects them downwards. If the magnitudes of the fields $E$ and $B$ are adjusted such that the force exerted becomes then the electron won’t feel any force.

Hence, we have the Electrostatic force equal to the Magnetic force.

$$F_E = F_L : qE = qvB$$

$$v = \frac{E}{B}$$
This phenomenon is used in construction of the velocity selector. The electrons moving with velocity $v$ travel in a straight path and emerge out through the slit R. Electrons having lesser velocity ($v' < v$) will get deflected upward along OP while those having velocities greater ($v'' > v$) will get deflected downward along OQ. These deflected electrons are absorbed by the screen.

Thus, a strictly homogeneous single velocity electron beam is obtained along OR.

**Cathode Rays:**

A highly evacuated discharge tube (pressure: $10^{-2}$mm to $10^{-3}$mm) is used for the production of cathode rays. The schematic diagram is as shown in the figure:

A negatively charged Cathode produces negatively charged electrons which are accelerated towards the positively charged Anode due to a high potential difference existing between them. This beam of negatively charged electrons obtained from the discharge tube is known as the cathode rays.

Some properties of Cathode rays are:

a) Cathode rays are emitted from the cathode surface normally irrespective of the position of the anode.
b) The rays travel in straight lines.
c) They cast shadows of objects on which they fall.
d) They produce heat on matter on which they fall.
e) They produce fluorescence.
f) They possess mechanical energy and hence exert mechanical pressure.
g) They can penetrate through small thickness of matter.
h) They ionize the gas through which they pass.
i) They affect photographic plates.
j) They are deflected by electric and magnetic fields.
k) When they strike solid substances, they produce X-rays.

**Canal Rays or Positive Rays:**

E Goldstein, while investigating the cathode rays, designed a discharge tube and discovered the Positive rays. The schematic apparatus is as shown below:
In the setup, a perforated cathode is kept in between the anode and a fluorescent screen. The electrons emitted by the cathode are accelerated towards the anode. On their way they collide with the atoms of the gas in the discharge tube. As a result electrons are knocked out of the neutral atoms and atoms become positively charged. This is known as ionization. These ionized ions are accelerated towards the cathode. Some of the ions pass through the hole in the cathode and strike the fluorescent screen. It is thus seen that the positive rays are positively charged ions of the gas contained in the discharge tube. As they pass through the canal in the cathode, they are also known as Canal rays.

Some properties of Positive rays are:

a) They are deflected by electric and magnetic fields. The deflection direction is opposite to that of the cathode rays.

b) They travel in straight line.

c) They produce phosphorescence and fluorescence.

d) They affect photographic plates.

e) They can cause sputtering in metals.

f) \(e/m\) is lesser than that of cathode rays due to higher masses of ions.

**Thomson’s Parabola Method for determination of \(q/m\) of positive ions**

J. J. Thomson, in 1911, developed a method of measuring the relative masses of different atoms by employing a parallel configuration of electric field \(E\) and magnetic field \(B\) simultaneously. The schematic is shown as below:

**Construction:** The experimental gas is filled into the evacuated glass discharge tube in which the gaseous discharge is produced. The perforated cathode \(K\) is made of a tube of soft iron faced with aluminium. Along the axial hole in this tube, there is a copper tube of fine bore of about 0.1 mm diameter through which the positive ions can pass. A strong electromagnet provides a magnetic field in the \(y\)-direction. The electric field is also set up in the same direction. Thus, parallel electric and magnetic fields are produced perpendicular to the axis of copper tube. A photographic plate acts as the screen and is placed at the end of the conical projection of the tube.

**Working:** The tube is evacuated to a pressure of about \(10^{-3}\) mm of Hg by means of a vacuum pump. The experimental gas is introduced at this pressure. A high voltage of about 20kV is applied between anode \(A\) and cathode \(K\). Cathode rays travelling from cathode to anode cause ionization of the gas and hence produces positive ions enroute. The positive ion beam travels toward cathode. After emerging from the cathode, the ions travel through the magnetic and electric fields. The deflected ions produce traces of parabolas on the photographic plate. The traces are produced on one side of the \(y\)-axis. The fields are reversed to obtain traces on other side of the \(y\)-axis.
The deflections produced by the positive ion having mass $m$, charge $q$ and velocity $v_x$ in the electric field $(y_E)$ and the magnetic field $(y_B)$ are given as,

$$y = y_E = \frac{DqIE}{mv_x^2} \quad \& \quad x = y_B = \frac{DqIB}{mv_x}$$

respectively.

Where, $l$ is the length of the horizontal path through the fields and $D$ is the distance of the screen from the centre of the fields. To eliminate velocity $v_x$, we square $x$ and divide it by $y$. Hence, we get

$$\frac{x^2}{y} = \left(\frac{DqIB}{mE}\right)^2 \frac{mv_x^2}{DqIE}$$

$$\frac{x^2}{y} = \frac{DqIB^2}{mE} = \frac{DLB^2}{E} \left(\frac{q}{m}\right)$$

Since, $D, l, B$ and $E$ are constants they can be replaced by a constant, $k$. Hence, we get

$$x^2 = k \left(\frac{q}{m}\right) y$$

This represents an equation of a parabola. The curves are as shown in figure,

The outermost line corresponds to hydrogen as $q/m$ for hydrogen is the highest. (Hydrogen ions are usually present due to the residual air in the tube.) The parabolas 1 and 2 correspond to the two isotopes of the gas introduced in the discharge tube. A line $y = a$ is drawn parallel to the x-axis. The x-coordinates $x_1, x_2$ and $x_H$ of its point of intersection with the parabolic traces are noted. So we get,

$$x_H^2 = k \left(\frac{q_H}{m_H}\right) a \Rightarrow \frac{q_H/m_H}{x_H^2} = \frac{1}{ka}$$

Similarly,

$$\frac{q_1/m_1}{x_1^2} = \frac{1}{ka}$$

$$\frac{q_2/m_2}{x_2^2} = \frac{1}{ka} \Rightarrow \frac{q_1/m_1}{x_1^2} = \frac{q_2/m_2}{x_2^2} = \frac{q_H/m_H}{x_H^2}$$

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Hence, we get

\[
\frac{q_1}{m_1} = \frac{x_1^2}{x_1^2} \left( \frac{q_H}{x_H^2 m_H} \right)
\]

\[
\frac{q_2}{m_2} = \frac{x_2^2}{x_2^2} \left( \frac{q_H}{x_H^2 m_H} \right)
\]

Thus, the values of \( q/m \) of the isotopes are found in terms of that of hydrogen.

If the ionic charge in each case is same, we get

\[
m_1 = \frac{x_1^2}{x_1^2} m_H
\]

\[
m_2 = \frac{x_2^2}{x_2^2} m_H
\]

Thus, we determine the masses of different isotopes of the desired element.

**Discovery of Isotopes:** In 1912, Thomson attempted to find the atomic weight of Neon using the parabola method. He discovered two parabolas, one corresponding to mass number 20 and another to 22. Thus, he found two kinds of neon atoms, identical in chemical nature and having the same optical spectra but different in mass. His co-worker, F. W. Aston later improved the results and along with the first two found an isotope of mass 21. Thomson method is an effective experimental tool which provides the firm experiment of the existence of stable isotopes of any desired element.

**Bethe’s Law:**

A non-uniform electric field is one in which the electric field varies from point to point. Electron motion in such fields is better understood with the help of equipotential surfaces. An equipotential surface is one such surface where the electric potential remains constant and the electric field lines are perpendicular to the surface at any point.

It may be thought of being set up by two metal plates charged to appropriate potentials. For allowing the passage of electrons, we think of an electric field set up by two closely spaced metal grids as shown in figure:
Region I has potential $V_1$ and region II has potential $V_2$. The plane surface AB constitutes one of the equipotential surfaces. Let an electron with velocity $v_1$ enter region I making an angle $I$ with the normal. As the electron passes through the equipotential surface AB, it experiences a force which alters its velocity. Because the electric field exists only in the $y$-direction, the vertical component ($y$-component) of electron changes while the tangential component ($x$-component) remains constant.

Thus,

$$v_{1x} = v_{2x}$$
$$v_{1} \sin I = v_{2} \sin r$$
$$\frac{\sin I}{\sin r} = \frac{v_{2}}{v_{1}}$$

If $V_1 > V_2$, $v_{1y}$ increases while if $V_2 > V_1$, $v_{2y}$ increases.

In our case we have taken $V_2 > V_1$. As the electrons move through the electric field their kinetic energy is provided by the respective potential energy of the electric fields.

Hence,

$$\frac{1}{2} m v_{1}^{2} = q V_1$$

and

$$\frac{1}{2} m v_{2}^{2} = q V_2$$

Dividing above equations we get,

$$\frac{v_{1}^{2}}{v_{2}^{2}} = \frac{V_1}{V_2}$$

$$\frac{v_{1}}{v_{2}} = \sqrt{\frac{V_1}{V_2}}$$

Hence we get,

$$\frac{\sin I}{\sin r} = \frac{v_{2}}{v_{1}} = \sqrt{\frac{V_2}{V_1}}$$

This is known as Bethe’s law of electron refraction. It is identical to the Snell’s law in optics. The difference here is that the ratio of electron velocities $v_1$ and $v_2$ are reversed.

When electron enters a region of higher potential, it accelerates and bends toward the normal while in case of light it decelerates and bends toward the normal.

**Electrostatic Lens:**

**Principle:** A stream of electrons experiences a change in direction of motion when it travels through a non-uniform field. Its path is bent at each equipotential surface. This is analogous to refraction of light through lenses. Thus, non-uniform electric fields can be used as lenses to converge or diverge a ray of electrons.

**Description:** A simple electrostatic lens comprises of two coaxial cylindrical metal tubes $T_1$ and $T_2$ of same size and separated by a small distance. $T_1$ is held at potential $V_1$ while $T_2$ is held at a potential $V_2$ higher than $V_1$. A non-uniform electric field is produced in the gap between the tubes due to the potential difference between the two tubes. The electric field lines are directed from tube $T_2$ to tube $T_1$. The
equipotential surfaces are perpendicular to the electric field lines everywhere. The electric field within the hollow space of the tube is weak and negligible. Such a schematic is shown below:

![Diagram showing equipotential surfaces and electric field lines.]

**Working:** Let us consider a thin bundle of electron rays entering the lens through tube T₁. For understanding the working of the lens, we will consider 3 electron rays. Electron ray 1 travelling through the axis and electron rays 2 & 3 travelling off axis.

Electron ray 1 on reaching the equipotential surfaces in the gap between the tubes experience electric force acting along the axis in forward direction. The electrons are therefore accelerated towards tube T₂ without any deviation from their path.

Electron ray 2 when reaching the equipotential surfaces experience electric force acting at an angle to the direction of their motion. The force F experienced by the electrons at the convex equipotential surface can be resolved into rectangular components $F_\parallel$ and $F_\perp$. $F_\parallel$ acts parallel to the axis while $F_\perp$ act perpendicular to the axis. Because of force $F_\perp$, the electrons are deflected down toward the axis while force $F_\parallel$ accelerates them forward. Thus, all the off-axis electrons try to converge toward the axis. However on crossing the midplane of the gap, the electrons encounter equipotential surfaces of concave shape. Force $F_\perp$ is directed away from the axis for all off-axis electrons while $F_\parallel$ is directed forward. As a result the electrons accelerate forward but tend to diverge. Thus, the first half of the lens acts as a converging lens while the second half acts as a diverging lens. For every set of voltages $V₁$ and $V₂$, the converging action is stronger than the diverging action. Moreover, the electrons travel slower in the first region owing to lesser potential while they travel faster in the second region. The net result is that the electron rays get focused. This arrangement is known as electrostatic lens.

**Electron Gun:**
An electron gun is a device which produces a narrow electric beam of high intensity. It makes use of the fact that non-uniform electric fields can be used to bend and subsequently focus electron rays. It was designed by V. K. Zworykin in 1933. The schematic is as shown below:

![Diagram of an electron gun showing its components and operation.]
**Construction:** The electron gun consists of a cathode K, filament heater F, grid G and two anodes A₁ and A₂. The cathode is a short hollow nickel cylinder and the filament is enclosed inside it. The front face of the cathode is coated with thoriated tungsten or barium and strontium oxides. The coating helps thermionic emission of electrons at temperatures of about 700°C to 900°C. The cathode is surrounded by the grid which is a negatively charged hollow metal cylinder having a small aperture to allow electrons to pass through. Anode A₁ and A₂ are kept beyond the grid. Both are short metal cylinders having central apertures and kept at a positive potential with A₂ at a higher potential than A₁. This whole assembly is kept in an evacuated space. A power supply provides the necessary voltages to the electrodes.

**Working:** When power is turned on, the filament heats up the cathode. At a temperature characteristic of the cathode, electrons are emitted from its front surface and they pass through the control grid. The grid is kept at a negative potential with respect to the cathode and controls the number of electrons passing through it. The grid with the negative potential on it acts as a gate and regulates the passage of electrons through it.

The cathode, the grid and the first anode A₁ constitute the first electron lens of the system. This lens is known as the pre-focussing lens. Electrons emitted from the cathode tend to diverge because they repel each other. The convex equipotential surfaces bend the electrons toward the axis. Consequently all electrons converge at a point P just inside the surface of the first anode. This point is known as the crossover point. The electrons emerging from P can easily be focused to a fine point. The role of the first lens is to converge the beam to the cross-over point which then acts as a point source of electrons for the second lens.

The anodes A₁ and A₂ form the second electron lens system which draws electrons from the cross-over point and brings them to a fine focus. The diaphragm D cuts off the wide angle electrons emerging from P. The focal point is controlled by adjusting the potentials on A₁ and A₂.

**Cathode Ray Tube:**

A Cathode Ray Tube (CRT) is a specially constructed vacuum tube in which an electron beam is controlled by electric or magnetic fields to generate a visual display of input electrical signals on a fluorescent screen. The schematic is as shown below:

![Cathode Ray Tube Schematic](image)

**Description:** The CRT resembles a horizontally placed conical flask sealed at its open end. The electron gun consisting of the filament, cathode, grid and the three anodes is mounted at one end of the tube as a single unit and electrical connections are given to them through base pins. The deflection system consists of two pairs of parallel plates oriented mutually perpendicular to the axis of CRT. The screen consists of a thin coating of phosphors deposited on the inner face of the wide end of the glass envelope. The inner surface is also coated with a conductive graphite coating called aquadag. It is internally connected to the accelerating anode.
**Working:**

i) **Electron Gun:** The indirectly heated cathode \( K \) emits a stream of electrons from its coated front face. The electrons pass through the control grid \( G \) held at a negative potential. The effective size of the aperture of grid depends on the potential difference between the grid and the cathode. The intensity of the glow on the screen is controlled by the number of electrons striking the screen. Therefore, by controlling the grid voltage, the number of electrons striking the screen can be controlled. Thus, the brightness of the spot can be controlled. The anodes \( A_1 \) and \( A_3 \) are internally connected and held at a higher positive potential of few kVs and \( A_2 \) is maintained at a relatively smaller positive potential. The anode \( A_1 \) accelerates the incoming electrons. The anode \( A_2 \) and grid \( G \) forms the first lens system which prefocuses the electron beam. Anode \( A_2 \) is called the focussing anode. The anodes \( A_2 \) and \( A_3 \) form the second lens system which focuses the electron beam to a fine point on the screen. The anode \( A_3 \) imparts further acceleration to the electron beam. By adjusting the potential of anode \( A_2 \) focus of the beam can be adjusted.

ii) **Deflection System:** The electrons emerging as a narrow beam from the electron gun speed towards the screen. On the way they pass through the deflection system. There are two types of deflection system:

a) Electrostatic type
b) Electromagnetic type

In the electrostatic type, two pair of metal plates is employed to deflect the electrons. One pair of plates is arranged horizontally near the anode \( A_3 \). When a potential difference is applied between the plates, an electric field is produced in the vertical direction to the axis. These plates are called as ‘vertical deflection plates’ or ‘Y-plates’. When a dc voltage is applied to Y-plates, the electrons get deflected vertically. Another pair of plates is arranged vertically to the axis. They produce an electric field in the horizontal direction. These plates are called as ‘horizontal deflection plates’ or ‘X-plates’. They deflect the electron beam in the horizontal deflection. When no voltages are applied to the deflection plates, the electron beam travels along the axis of the CRT and strikes the centre of the screen. By varying the voltages on both the plates, the luminous spot can be moved to any position in the plane of the screen.

iii) **Fluorescent Screen:** The interior surface of the circular front face of the CRT is coated with a thin translucent layer of phosphors. The phosphor coating glows at the point where it is struck by the electron beam.

iv) **Aquadag:** Electrons impinging on the screen tend to charge it negatively and hence causing repelling of electrons coming afterwards. This will result in decrease of electrons reaching the screen and hence the brightness of the glow reduces.

The electrons striking the screen not only cause emission of light but also produce secondary emission of electrons.

The cathode gradually assumes a positive charge as electrons are emitted from it in large numbers.

Hence, the aquadag coating is used to remove the excess electrons and returned to the cathode via ground.

**Limit of Electrostatic Deflection:**

We know that the deflection sensitivity of an electron in an electric field is given as

\[
S = \frac{1D}{2dV_A} \text{ m/Volt} = \frac{5001D}{dV_A} \text{ mm/Volt}
\]

**Total spot deflection** = **Deflection sensitivity** x **Applied voltage between the plates**
The angle of deflection of the electron beam in the electrostatic type CRT is given by:

\[ \theta = \tan^{-1} \left( \frac{I_D}{2dV_A} \right) \]

Theoretically, a large angular deflection can be obtained by selecting longer deflection plates (larger \( l \)) and keeping them closer (smaller \( d \)). But, in practice, the deflection \( \theta \) has to be restricted to smaller values. Otherwise, the electrons might strike the deflection plates instead of the screen. Because of the restricted angular deflection, the area that can be covered by the electron beam on the screen becomes smaller. It is thus not possible to obtain bigger displays using electrostatic deflection.

Cathode Ray Oscilloscope:

A cathode ray oscilloscope (CRO) is used to measure electrical signals, time intervals and phase shift between two electrical signals. The internal sections of an oscilloscope contain the following seven major sections:

a) Cathode ray tube (CRT)  
b) Time base circuit  
c) Trigger circuit  
d) Vertical circuits  
e) Horizontal circuits  
f) High voltage power supply  
g) Low voltage power supply

The arrangement of these sections in a CRO is shown below:

---

i) CRT: A cathode ray tube with electrostatic deflection forms the central part of CRO. The high voltage power supply section provides the required high potentials to the various electrodes of CRT. The CRT generates the electron beam, focuses it and accelerates it toward the fluorescent screen. The rest of the sections are electronic circuits which cause the desired movement of the luminous spot on the screen.

**Vertical Movement:** In the field of electronics, the message is an electric signal. This signal is applied to the Y-plates. When a signal voltage is applied to Y-plates, the polarity and magnitude of voltage on the plates vary with each alternation of the cycle. As a result, the luminous spot moves up and down on the screen at the same frequency as that of the signal. The phosphors continue to glow for a short time after the electron beam passes, and due to the persistence of vision, the path of the beam across the screen is seen as a vertical line. It is called the trace. The length of the vertical trace corresponds to the peak to peak voltage \( V_{p-p} \) of the applied signal. The details of the signal are not revealed through these vertical motions. The signal form can be known only when the beam moves horizontally from left to right.

**Horizontal Movement:** The horizontal motion of the electron beam is produced when an ac signal is applied to the X-plates. In a signal the voltage varies as a function of time. The electron beam gets deflected through equal distances per unit time when the voltage applied to X-plates increases through equal amounts of voltage per unit time. Also, at the end of the horizontal motion, the beam returns to the starting point.
ii) **Time Base Circuit:** To display the signal applied to the Y-plate, it has to be horizontally moved or swept by the voltage applied to the X-plates. A ramp voltage also known as sawtooth voltage fulfills this requirement. Such kind of waveform is as shown below:

![Waveform Diagram](image)

A time base circuit mainly consists of a time base generator. The time base generator is a variable frequency oscillator which produces the above mentioned ramp voltage.

Ideally the voltage increases uniformly with time from a negative value \( (V_x)_\text{min} \) to a maximum value \( (V_x)_\text{max} \) and from there suddenly dips to minimum.

**Display of Signal:** As the signal is applied to Y-plates and time base voltage to the X-plates, the electron beam is subjected to two forces acting in perpendicular direction to each other. The deflection of the beam at any instant occurs along the direction of the resultant of the two forces. As time progresses, the resultant change in magnitude and direction and the beam displays the actual shape of the signal on the screen. The process is tabulated below:

<table>
<thead>
<tr>
<th>Position of Y-signal</th>
<th>Position on screen</th>
<th>Position of sawtooth voltage</th>
<th>Position on screen</th>
<th>Position of resultant (output) on screen</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (On left extreme of X-axis)</td>
<td>Centre of X-axis</td>
<td>1 ((V_x)_\text{min})</td>
<td>Left extreme on the X-axis</td>
<td>Left extreme on the X-axis</td>
</tr>
<tr>
<td>2 (maximum in positive Y-direction)</td>
<td>Top of the Y-axis</td>
<td>2 (In between ((V_x)_\text{min}) and axis)</td>
<td>In between left extreme and centre of the X-axis</td>
<td>Above the X-axis in the second quadrant corresponding to (V_p)</td>
</tr>
<tr>
<td>3 (Centre of X-axis)</td>
<td>Centre of X-axis</td>
<td>3 (Centre of X-axis)</td>
<td>Centre of X-axis</td>
<td>Centre of X-axis</td>
</tr>
<tr>
<td>4 (maximum in negative Y-direction)</td>
<td>Bottom of the Y-axis</td>
<td>4 (In between ((V_x)_\text{max}) and centre of X-axis)</td>
<td>In between right extreme and centre of X-axis</td>
<td>Below the X-axis in the first quadrant corresponding to (V_p)</td>
</tr>
<tr>
<td>5 (On right extreme of X-axis)</td>
<td>Centre of X-axis</td>
<td>5 ((V_x)_\text{max})</td>
<td>Right extreme on the X-axis</td>
<td>Right extreme on the X-axis</td>
</tr>
</tbody>
</table>

![Diagram](image)

After this the beam returns to position 1 and the process starts for one more cycle.
iii) **Trigger Circuit:** To display a stationary wave pattern on the CRO screen, the horizontal deflection should start at the same point of the input signal in each sweep cycle. When this happens we say that the horizontal sweep voltage is synchronized with the input signal. If the sweep and the signal voltages are not synchronized, a still pattern is not displayed.

One of the methods to achieve synchronization is using a trigger circuit. In this method, a part of the output obtained from the vertical amplifier is fed to a trigger generator. Trigger generator is sensitive to the level of voltage applied at its input. When triggered, it produces a pulse which is fed to the time base generator and it starts one sweep cycle of the time base voltage.

The signal is synchronized when its frequency equals the sweep frequency or an integral multiple of the sweep frequency. That is,

\[
F_{\text{signal}} = n F_{\text{sweep}} \\
T_{\text{sweep}} = n T_{\text{signal}}
\]

As an example suppose \( T_{\text{sweep}} = T_{\text{signal}} \), one wave will be displayed on screen. While if \( T_{\text{sweep}} = 2 T_{\text{signal}} \), two waveforms will be displayed.

iv) **Vertical Circuits:** The vertical circuits mainly consist of an attenuator and a voltage amplifier. The signal to be tested is applied at the Y-input. The signal amplitude is increased or decreased by changing the amount of attenuation and then fed to the amplifier so that adequate deflection is obtained on screen.

v) **Horizontal Circuits:** The horizontal circuits also consist of an attenuator and a voltage amplifier. When the sweep selector switch is in ‘INT’ position, the sweep voltage from the time base generator is applied to the amplifier while when it is at ‘EXT’ position; the horizontal input is disconnected from the internal sweep generator and connected to the X-input.

vi) **Low Voltage Power Supply:** The low voltage power supply provides power to electronic circuits like amplifiers, trigger generator, time base generator. Its output is in the order of few tens to few hundreds of volts.

vii) **High Voltage Power Supply:** The low voltage power supply provides power to the electrodes in the CRT. The voltages are of the order of 1600 to 2200 volts.

**Applications of CRO:**

i) **Study of Wave Forms:** CRO is widely used to study waveforms from different electronic circuits. The signal under study is applied at the Y-input while X-plates are internally connected to the time base generator. The wave form is displayed on the screen.

ii) **Measurement of dc voltages:** Initially trace is adjusted to the centre of the screen. The dc voltage under study is applied at the Y-input. The trace gets deflected upward or downward depending upon the polarity of the applied voltage. The deflection is noted and multiplying it with the deflection factor (volts/cm), the magnitude of unknown voltage is obtained.

iii) **Measurement of ac voltages:** The trace is adjusted to the centre and the voltage under study is applied at the Y-input. The peak to peak distance is measured and on multiplying with the deflection factor, \( V_{p-p} \) is obtained. The rms and the average value of the voltage can be calculated by:

\[
V_p = V_{p-p}/2
\]
\[ V_{rms} = \frac{V_p}{\sqrt{2}} \]

\[ V_{ave} = 0.636 V_p \]

**Measurement of Frequency:** A sinusoidal voltage whose frequency is to be determined is applied to the Y-input. The time base control is adjusted to obtain 2 or 3 cycles of the signal on screen. The horizontal spread of one cycle is noted. By multiplying with the time base sensitivity, the time period of the signal is obtained. The reciprocal of the time period gives the frequency of the signal.

**Lissajous Figures:** Another method that can be employed to determine the frequency is the use of Lissajous patterns. When two sine waves oscillating in mutually perpendicular planes are combined, different types of closed loop patterns are obtained. The form of the resultant pattern depends on the ratio of frequencies and phases of the two waves. Lissajous patterns can be obtained on a CRO by applying two sine wave voltages at Y and X plates simultaneously.

To find the unknown frequency of a signal, it is applied to the Y-plates while a sine wave voltage of known frequency is applied to the X-plates. By varying the frequency of the known source a stable loop pattern is displayed. Horizontal and vertical tangents are drawn. The number of points at which the loops are touching the horizontal (\(L_H\)) and the vertical (\(L_V\)) tangents are noted. The unknown frequency is given by:

\[ f_y = f_x \left( \frac{L_H}{L_V} \right) \]

One such figure is shown below:

\[ \text{iv) Measurement of Phase Difference:} \] When two sine waves oscillating in mutually perpendicular planes are of the same frequency, the Lissajous pattern takes the form of an ellipse. The ellipse helps in determining the phase relationship between two identical sine waves.

Let two sine waves of same amplitude and same frequency are applied. Then,

\[ V_x = V \sin \omega t \]
\[ V_y = V \sin (\omega t + \phi) \]

Where, \(\omega\) is the angular frequency and \(\phi\) is the phase difference.

The ellipse obtained is as shown below:

Note down AA’ and BB’. The phase angle is given by:

\[ \phi = \sin^{-1} \left( \frac{AA'}{BB'} \right) \]

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**Obtaining Different Lissajous Figures on CRO:**

If \( v_x \) and \( v_y \) are the instantaneous values of the voltages applied to the X and Y plates, they can be expressed as:

\[
\begin{align*}
v_x &= V_x \sin \omega_x t \\
v_y &= V_y \sin (\omega_y t + \phi)
\end{align*}
\]

Where, \( V_x \) and \( V_y \) are amplitudes of voltages applied, \( \omega_x \) and \( \omega_y \) are angular frequencies of the voltages and \( \phi \) is the phase angle.

By varying the above quantities, various patterns can be obtained.

i) When both voltages have equal angular frequency and are in phase with each other.
\[
\omega_x = \omega_y \text{ and } \phi = 0
\]
\[
\Rightarrow v_y = \left( \frac{v_x}{v_y} \right) v_x
\]

This is the equation of a straight line. Thus, the pattern obtained is a straight line.

ii) When both voltages have equal angular frequency but with \( \pi/2 \) phase difference.
\[
\omega_x = \omega_y \text{ and } \phi = \pi/2
\]
\[
\Rightarrow \frac{v_x^2}{v_x^2} + \frac{v_y^2}{v_y^2} = 1
\]

This is the equation of an ellipse. Thus, the pattern obtained is an ellipse.

iii) When two equal voltages have equal angular frequency but with \( \pi/2 \) phase difference.
\[
\omega_x = \omega_y \text{ and } \phi = \pi/2 \text{ with } V_x = V_y = V
\]
\[
\Rightarrow v_x^2 + v_y^2 = V^2
\]

This is the equation of a circle. Thus, the pattern obtained is a circle.

iv) If the angular frequencies are in ratio \( n_x/n_y \) where \( n_x \) and \( n_y \) are simple integers
\[
\frac{\omega_x}{\omega_y} = \frac{n_x}{n_y}
\]

Then closed figures with multiple loops are obtained.

If \( n_x = 1 \) and \( n_y = 2 \), figure of eight is obtained.

The figures are as shown below:

![Fig i)](image1)

![Fig ii)](image2)

![Fig iii)](image3)

![Fig iv)](image4)
Uses:

i) It is used in troubleshooting or circuit analyzing in radio engineering, electronic industry etc.
ii) It is used for creating images in a television receiver.
iii) It is used in hospitals in electrocardiography, ultrasonography etc.
iv) It is used in radar to detect and give position, angle, distance etc. of a target.

Magnetic Field Focussing:

Longitudinal Magnetic Field Focussing:
It employs uniform magnetic field acting along the direction of motion of the electron beam. The path of the electron in a uniform magnetic field would be a helix if the electrons enter the field at an angle. The pitch of the helical path is given by:

\[ p = \frac{2\pi mv\cos\theta}{qB} \]

Let us consider a bundle of electron rays of a given velocity \( v \) enter a solenoid making different angles with the magnetic field. If the divergence in the beam is assumed to be small (\( \theta < 10^\circ \)), \( \cos\theta \) can be taken to be unity. It is also known that the time period of an electron entering the field remains constant. Thus, any electron entering the field intersects the field lines after the time interval \( T \). Therefore, the electron rays entering at small angles at \( O \) will again cross the same field line at a common point \( F \) after the time period \( T \). The distance from \( O \) to \( F \) is \( l \) and is given by:

\[ l = \frac{2\pi mv}{qB} \]

Thus, a longitudinal homogeneous magnetic field focuses electrons at a distance \( l \) or at any distance which is an integral multiple of \( l \) (i.e. \( nl \)).

Transverse Magnetic Field Focussing (180° Focussing):

A transverse uniform magnetic field is employed to focus a beam of charged particles. The focussing would be affected after the beam rotates through an angle of 180°. If a beam of positively charged particles with the same velocity enter a transverse uniform magnetic field and if the angular divergence, \( \phi \), is very small, it gets focused after completing a semicircle. The radius of curvature is given by:

\[ r = \frac{mv}{qB} \]

If isotopes are used, owing to different masses, beams of isotopes would have different radii of their respective semicircles and would be focused at different points as shown in figure.
The particles having the same momentum, $mv$, will get focuses at same point. This is called the momentum selector principle.

In fig. b, beams 1 and 3 are focused at P and beam 2 at Q. The distance PQ is given as,

$$\sqrt{g_{0862}}\sqrt{g_{0863}} \sqrt{g_{3606}} 2\sqrt{g_{0870}}\sqrt{g_{6666}} 1\sqrt{g_{3398}}$$

For small $\phi$, $cos \phi = 1 - \frac{\phi^2}{2!} + \frac{\phi^4}{4!} \ldots \ldots = 1 - \frac{\phi^2}{2!}$ approximately

Hence, $PQ = 2r \left( 1 - \left( 1 - \frac{\phi^2}{2!} \right) \right) = r \phi^2$

This method is very effective in separating isotopes and was utilized by Ernest Rutherford in 1914 for the first time.

**Magnetic Lens:**

Magnetic fields which are symmetric along the axis (Longitudinal fields) have a focusing effect on an electron beam passing through them. By encasing the coils in hollow iron shields the magnetic fields are concentrated and improved focussing action is obtained. Such solenoids are called as thin magnetic lenses. It is known that an electron travelling in a non-uniform magnetic field describes a helical path. The radius of loop is given by

$$r = \frac{mv}{qB}$$

If particles of same mass having same charge and same velocity enter a non-uniform field then the radius of the loop ($r$) inversely depends on magnetic field $B$. Thus, if the electron moves into stronger magnetic fields, the radius of the loops goes on decreasing. Thus in a solenoidal field, the helical path of the electron is twisted into tighter loops and the turns become smaller and smaller. Ultimately all the electrons converge at the point of focus on the axis as shown in figure a.

In the second magnetic lens, electrons entering the field at P get focused at Q.

The focal length of the magnetic lens can be adjusted by suitably changing the strength of the magnetic field. This type of magnetic lens is used to design advanced electron microscopes.
**Bainbridge Mass Spectrograph:**

Mass spectrograph is an instrument which, by using electric and magnetic fields, separates different isotopes from a stream of positive ions of an element and measures their individual masses. In 1933, K. T. Bainbridge designed a mass spectrograph in which a velocity selector was used to produce a mono-velocity ion beam and a transverse magnetic field was employed to distinguish between ions of different masses.

**Principle:** The spectrograph is based on the principle that a non-uniform magnetic field acting perpendicular to the path of ions having the same velocity deflects the ions of different masses from a straight path into several circular paths of different radii. The radius is proportional to the mass of the ion describing the circular path.

**Construction:** It consists of a vacuum chamber placed in a uniform magnetic field acting perpendicularly to its larger surface. Slits $S_1$ and $S_2$ collimate incoming ion beam. The deflection plates are placed next to the slits. The electric field produced by the charged deflection plates and the transverse magnetic field constitute a velocity filter. Slit $S_3$ further collimates the mono-velocity ion beam. A photographic plate is mounted in the analyzing chamber in line with the slit $S_3$. The schematic is as shown below:

![Diagram of Bainbridge Mass Spectrometer](image)

**Working:** The element to be studied is taken in the form of gas is introduced into a discharge tube. The gas is ionized and positive ions enter the mass spectrograph through slit $S_1$ and $S_2$. The ions in the beam have a wide range of velocities as they pass through the velocity filter. The fields are adjusted such that the ions come out with velocity $v = E/B$ and pass through slit $S_3$. The ions enter a magnetic field $B'$ having field direction perpendicular to the direction of motion of ions. The ions describe circular paths and are recorded on the photographic plate.

**Theory:** Consider a beam of ions having charge $q$ but having different velocities enter the velocity selector. In the velocity selector, electric field $E$ and magnetic field $B$ are applied perpendicular to each other. The velocity of electrons coming out of it is given by:

$$v = \frac{E}{B}$$

The radius of the circular paths described by the ions is given by

$$r = \frac{mv}{qB'}$$
In the mass spectrograph velocity, charge and magnetic field are all constant at a given time. Thus, the radius will only depend on the mass of the ions. Therefore ions of higher masses will have larger radii and vice-versa.

We have,

\[ qvB' = \frac{mv^2}{r} \]

\[ m = \frac{qB'r}{v} = \frac{qB'B}{E} r \]

Also, the difference between two radii can be found as:

\[ \Delta r = (r_2 - r_1) = \frac{E}{qB'B}(m_2 - m_1) \]

**Particle Accelerators:**

To study nuclear reactions, charged particles accelerated to very high energies are required. The devices which impart such energies are called as particle accelerators. Two types of particle accelerators were developed during 1931-32.

a) **Linear Accelerator:** In a linear accelerator, the charged particles are allowed to accelerate through a large potential difference \( V \) maintained between two electrodes. However it is difficult to produce and maintain voltages greater than a few hundred kilo volts. This limits the maximum energy achievable to be a fraction of an MeV.

b) **Cyclic Accelerator:** E.O. Lawrence and M. S. Livingston at the University of California built an accelerator in which charged particles are given small doses of voltages a number of times and hence obtaining the required energy. They called their accelerator as cyclotron.

**Linear Accelerator (Linac):**

A linear accelerator (or a Linac) is a device which accelerates charged particles in a straight line by means of oscillating electric field that provides a series of steady accelerating steps in correct phases at a series of gaps between the electrodes.

**Principle:** In a linear accelerator, a moderator accelerating potential is applied a number of times so that the charged particles are accelerated along a straight line.

**Construction:** A simple schematic of a linear accelerator is as shown below:

![Linear Accelerator Diagram](image)

The charged particles or ions are emitted by the source and travel along the axis of a series of coaxial cylindrical electrodes 1,2,3,4, etc. These electrodes are called the drift tubes. The drift tubes are connected to an AC source of high frequency (say a High Frequency Oscillator) so that alternate tubes...
have potentials of opposite sign. Thus in one half-cycle if tubes 1 and 3 are positive, 2 and 4 will be negative. After every half cycle, polarities are reversed.

**Working:** Suppose a positive ion, having charge \( q \) and mass \( m \), is accelerated during the half cycle when the drift tube 1 is negative. If the potential applied to the drift tube is \( V \), then velocity \( v_1 \) of the ion on reaching the drift tube is given by

\[
\frac{1}{2}mv_1^2 = qV
\]

\[\Rightarrow v_1 = \frac{2qV}{m}\]

The ions are accelerated in the gap between the tubes but travel with constant velocity in the field free space inside the tubes. The length of the tube 1 is so adjusted that when the positive ions come out of the tube, the tube has a positive potential and the next tube i.e. tube 2 has a negative potential. As the polarities change, the positive ion is again accelerated in the space between the tubes 1 and 2. The velocity of the ion on reaching tube 2 is give as

\[
\frac{1}{2}mv_2^2 = 2qV
\]

\[\Rightarrow v_2 = \sqrt{2} \frac{2qV}{m} = \sqrt{2}v_1\]

This shows that \( v_2 = \sqrt{2} \) times \( v_1 \). The ion has to travel with the same time as it had done in tube 1 so that when it comes out of tube 2, the polarities of the tubes should have reversed to facilitate acceleration. As the velocity \( v_1 \) is \( \sqrt{2} \) times the velocity \( v_1 \), the length of tube 2 should be \( \sqrt{2} \) times that of tube 1. For successive accelerations in the gaps the tubes 1,2,3,4, etc must have lengths proportional to 1,\( \sqrt{2} \),\( \sqrt{3} \),\( \sqrt{4} \), etc. to a first approximation.

**Energy of the ion:** If \( n \) is the number of gaps that the ion has to travel in the accelerator and \( v_n \) is the final velocity acquired by it then,

\[v_n = \sqrt{n} \frac{2qV}{m}\]

Hence, Kinetic energy of the ion is given as, \( K.E. = \frac{1}{2}mv_n^2 = nqV\)

The final energy of the ions when they strike the target depends upon the overall length of the accelerator i.e. the total number of gaps and energy gained in each gap. The beam striking the target comprises of pulses of particles. The number of pulses per second is equal to the frequency of the alternating potential applied to the tubes.

**Length of the tubes:** The time taken by the ion to travel through a drift tune is equal to the half of the time period \( (T) \) of the high frequency oscillator voltage. This will facilitate that when the ion comes out of a drift tube, the polarity of the next tube will be reversed.

If \( v_n \) is the velocity of the ion, the length of \( n \)th tube is \( l_n \) and the frequency of the oscillator is \( f \), then the time of passage through \( n \)th tube is given as,

\[t = \frac{l_n}{v_n} = \frac{T}{t} = \frac{1}{2f}\]

\[\Rightarrow l_n = \frac{v_n}{2f} = \frac{1}{2f} \frac{2qV}{m}\]
Limitations:

i) The length of the accelerator becomes inconveniently large and it becomes difficult to maintain vacuum in such a large chamber.

ii) The ion current is available in the form of pulses of short duration.

Cyclotron:

**Principle:** A charged particle moving in a transverse uniform magnetic field describes a circular path. The frequency of the revolution of the particle is given by

\[ f = \frac{qB}{2\pi m} \]

It is seen that the frequency is independent of the particle velocity and hence its kinetic energy. Thus, slower particles will move in smaller circles and vice-versa so as to take same time to complete one revolution. Hence, charged particles having different initial velocities can be accelerated to produce a high energy beam even by a moderate potential difference. The particle has to go through the electric field due to the action of a uniform magnetic field applied normal to the particle path. The particles gain small amount of energy in each cycle and the final energy is the product of the number of cycles and energy gained in each cycle.

**Construction:** A short cylindrical hollow metal box is cut into two halves along a diameter. The resulting semicircular chambers resemble the letter ‘D’ and hence are called as ‘dees’. The dees are separated by few centimetres from each other and constitute the electrodes. They are insulated from each other and are enclosed in a vacuum chamber. A powerful electromagnet produces a magnetic field perpendicular to the direction of the velocity of the electrons. A high frequency oscillator is connected to the dees which establish an electric field within the gap between the dees. An ion source is located near the centre of the dees. The schematic of a cyclotron is as shown below:

![Diagram of a cyclotron]

**Working:** Let us suppose the ion source emits positively charged ions. Let at the same time, the dee D$_1$ is at negative potential. The ions will be attracted towards dee D$_1$. The electric field is mainly concentrated across the gap and negligible in the dees. The transverse magnetic field makes the ions go in a circular path. The time period of the ions in the magnetic field is given by

\[ T = \frac{2\pi m}{qB} \]
So, the ions arrive at the gap between the dees completing half the revolution in time $T/2$. If the frequency $f_0$ of the oscillator is adjusted such that when the ions reach the gap, the potential in the dees get reversed and now dee $D_2$ is negatively charged and the ions are attracted towards it. In each revolution, the ions gain energy $2qV$, where $V$ is the potential applied by the oscillator. As the ions gain energy, their velocities increase. As the radius of charged particles depend on the velocities in a uniform magnetic field, the radius of the ion paths in the dees increases at each revolution. After about hundred or more revolutions, the ions acquire energies of the order of several MeVs. At the end of the journey, the ion beam passes through a narrow slit in side wall of one of the dees and moves through the dc electric field of a capacitor constituted by the dee wall and a deflector plate. The emerging beam is used to bombard on a target under study.

The maximum velocity ($v_m$) obtained from the cyclotron is given as

$$v_m = \frac{r_m qB}{m}$$

Using, $r = \frac{mv}{qB}$

Therefore, the maximum kinetic energy obtained is

$$K_{\text{max}} = \frac{1}{2}mv_m^2 = \frac{r_m^2 q^2 B^2}{2m} \text{ Joules}$$

**Limitations:**

i) It is seen that the oscillator frequency is given by

$$f_0 = \frac{qB}{2\pi m}$$

Thus, higher the cyclotron oscillator frequency, stronger the magnetic field required. This results in increase in the maximum energy of the ions. But there is a limit to the maximum strength of magnetic field that can be produced. And hence there is an upper limit to energy of the ions coming out of the cyclotron.

ii) A more basic limitation arises due to relativistic variation of the mass of the accelerated particle. As a particle approaches the speed of light, its mass increases according to the relation

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Correspondingly, the time period of the particle inside the dees becomes:

$$T = \frac{2\pi}{qB} \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Thus, with the increase in the value of the velocity, the time ($T$) required to reach the gap increases. Hence, the ions lag behind the applied oscillator voltage. This results in stopping the acceleration of the ions.

To prevent such a condition, the ion frequency is kept constant by maintaining the value of

$$B \sqrt{1 - \frac{v^2}{c^2}}$$

constant. This means $B$ is made to increase with the help of a strong electromagnet. Such type of a modified version of cyclotron is called as synchrotron.
Formula Time

1) Charge on an electron: \(-1.6 \times 10^{-19}\) C
2) Mass of an electron: \(9.1 \times 10^{-31}\) Kg
3) Electric Field: \(E = F/q = V/d\)
   Where, \(V =\) Potential difference \(d =\) Distance between the plates
   \(F =\) Electric Force \(q =\) Charge
4) Electric Potential: \(V = W/q\)
   Where, \(W =\) Work done
5) Force on a current carrying conductor in a magnetic field: \(F = Blis\sin\theta\)
   Where, \(B =\) magnetic field \(l =\) Current through the conductor
   \(I =\) length of the conductor \(\theta =\) Angle between the direction of current and the direction of the magnetic field
6) Force on a moving charge in a magnetic field: \(F = Bqv\sin\theta\)
   Where, \(q =\) charge on the moving particle \(v =\) velocity of the particle
   \(\theta =\) Angle between the direction of motion of charge and the direction of the magnetic field
7) Lorentz Force: \(F = q(E + v \times B)\)
8) Acceleration of electron in electric field: \(a = \frac{F}{m} = \frac{eE}{m} = \frac{eV}{dm}\)
9) Velocity of electron at any given time in an electric field: \(v = \frac{eEt}{m} = \frac{eV}{md} = \sqrt[2]{\frac{2eES}{m}} = \sqrt[2]{\frac{2eVS}{md}}\)
10) Terminal velocity: \(v = \sqrt[2]{\frac{2eV}{m}}\)
11) Y- deflection of an electron in an electric field acting perpendicular to its motion:
    \(y_p = \frac{eE}{2mv_y^2} x^2 = \frac{eV}{2mdv_y^2} x^2\)
12) Y- deflection of an electron outside an electric field acting perpendicular to its motion:
    \(y = \frac{1}{2} \frac{DV}{dV_a}\)
    Where, \(D =\) Distance between middle of the plate and the screen
    \(l =\) length of the plates
    \(V_a =\) Accelerating voltage
13) Electrostatic deflection sensitivity: \(S = \frac{y}{V} = \frac{10}{2dV_a}\)
14) Electrostatic deflection factor: \(G = \frac{2dV_a}{10}\)
15) Radius of the path of an electron executing circular motion in a transverse magnetic field: \(r = \frac{mv}{eB}\)
16) Time period of the path of an electron executing circular motion in a transverse magnetic field:
    \(T = \frac{2\pi r}{v} = \left(\frac{2\pi}{v}\right)\left(\frac{mv}{eB}\right) = \frac{2\pi m}{eB}\)
17) Pitch of the helical path in which an electron moves if it is subjected to move in a magnetic field with its velocity vector making an angle \(\theta\) with the axis: \(p = \frac{2mv\cos\theta}{eB}\)
18) Magnetic deflection sensitivity: 
\[ S_M = \frac{y}{B} = Dl \sqrt{\frac{e}{2mv_0}} \]

19) Velocity of electrons in mutually perpendicular electric and magnetic field: 
\[ v = \frac{E}{B} \]

20) Thomson’s parabola method: 
\[ \text{a) For } \frac{q}{m} = \frac{x^2}{x_0} \left( \frac{q\mu}{m_H} \right) \]
\[ \text{b) For mass: } m = \frac{x^2}{x_0^2} m_H \]

21) Bethe’s law: 
\[ \frac{\sin \theta}{\sin \phi} = \frac{v_2}{v_1} = \sqrt{\frac{V_2}{V_1}} \]

22) Determination of unknown frequency by Lissajous patterns: 
\[ f_y = f_x \left( \frac{L_H}{L_V} \right) \]

23) Determination of phase difference: 
\[ \phi = \sin^{-1} \left( \frac{A A'}{B B'} \right) \]

24) Determination of mass using mass spectrograph: 
\[ m = \frac{qB'r}{v} = \frac{qB'B}{E} r \]

25) Velocity of ion from \( n \)th tube: 
\[ v_n = \sqrt{n} \frac{2gV}{\sqrt{m}} \]

26) Maximum Kinetic energy: 
\[ K.E. = nqV \]

27) Length of \( n \)th tube: 
\[ l_n = \frac{v_n}{2f} = \frac{1}{2f} \sqrt{\frac{2nqV}{m}} \]

28) Cyclotron frequency: 
\[ f_0 = \frac{qB}{2\pi m} \]

29) Maximum kinetic energy obtained from a cyclotron: 
\[ K_{max} = \frac{1}{2} m v_m^2 = \frac{r_m^2 q^2 B^2}{2m} \text{ joules} \]

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